CK-12 FOUNDATION

Measurement in Chemistry

Parsons

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Chapter 1

Measurement in Chemistry

1.1 Making Observations

Lesson Objectives

The student will:

- explain the importance of observations.
- define qualitative and quantitative observations.
- distinguish between qualitative and quantitative observations.
- explain the importance of using quantitative observations in measurements where possible.

Vocabulary

- observation
- qualitative observation
- quantitative observation

Introduction

Observation is very important when using scientific methods to investigate phenomena. **Observation** involves using the senses to gather information about the natural world. Science depends on keeping records of observations for later interpretations. These interpretations may lead to the development of scientific theories or laws. Without accurate observations, scientists cannot make any interpretations and therefore cannot draw conclusions.

Take out a piece of paper and record a chart similar to the one shown in **Table** 1.1. A chart is a useful tool that can help us record and organize our observations. Look up from this text and scan the room. Write down what you see around you in as much detail as you feel necessary in order for you or someone else to picture the room.

Table 1.1: **Record of Observations**

Item	Observation
1.	

Table 1.1: (continued)

Item	Observation
2.	
3.	

One summer evening, Scott and Brenda came home from work to find their house in shambles. Neighbors, friends, and colleagues were baffled by the strange occurrence. Inside the house, they found the television turned on. Food on the table was ready to be eaten. All of Scott's coin collections, his precious metals, and Brenda's prized possession – her statue of Galileo – were gone. Foul play is suspected.

Here is a simple test for you. Pretend you are visiting a forensic scientist, hired to investigate the scene of the crime. You are asked to only analyze the observations gathered by the other investigators at the scene. You must try to make as few assumptions as possible and make your decision based on the data at hand. The lead investigator gives you the following observations gathered from the scene and the suspects:

Observations at Scene

1. Blood type = B
2. Fiber sample = polyester
3. Powder found = white
4. Shoe print found = work boot

Table 1.2: **Suspect Information**

Suspect 1: 180 lb male	**Suspect 2: 220 lb male**	**Suspect 3: 120 lb female**
Blood type = B	Blood type = B	Would not comply
Sweater = polyester	Blazer = wool knit	Pants = polyester
Works in sugar factory	Pastry chef	Automobile sales woman

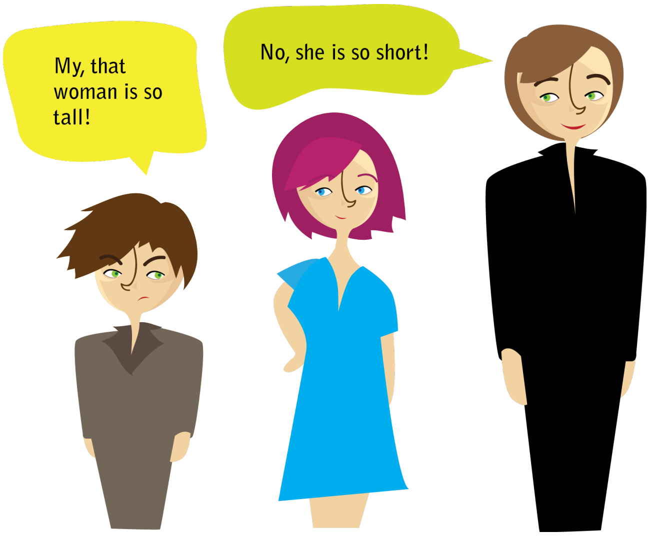
From **Table** 1.2, can you deduce who might have been involved in the alleged crime? Do you need more information? How detailed do the observations need to be in order for you, the scientist, to make accurate conclusions? What will you base your decision on? What other information do you need? Remember that if you guess randomly, an innocent person could be convicted.

Quantitative and Qualitative Observations Defined

There are two types of observations: quantitative and qualitative. **Quantitative observations** involve measurements or estimates that yield meaningful, numerical results. **Qualitative observations** yield descriptive, nonnumerical results. Although all the observations we can make on a phenomenon are valuable, quantitative observations are often more helpful than qualitative ones. Qualitative observations are somewhat vague because they involve comparative terms. Quantitative observations, on the other hand, have numbers and units associated with them and therefore convey more information. Even an estimated number is more valuable than no number at all.

A qualitative observation would be, for example, "The attendance clerk is a small woman." If the observer is 6 feet 4 inches tall, he might find a woman who is 5 feet 8 inches tall to be "small." If the observer reported this to someone who is 5 feet 2 inches tall, however, the listener would not acquire a good idea

of the attendance clerk's height because he would not think a woman who is 5 feet 8 inches tall is small. The description "a small woman" could refer to any woman whose height was between 3 feet and 6 feet, depending on who made the observation, as illustrated in the image below.



Similarly, "a small car" could refer to anything from a compact car to a child's toy car. The word "small" is a comparative term. The same is true for all words like tall, short, fast, slow, hot, cold, and so forth. These words do not have exact meanings. Under various circumstances, temperatures of 90°F, 110°F, 212°F, and 5000°F could all be described as "hot." The word "hot" does not convey as much information as the numerical description. Even observations of colors are not exact because there are many shades of each color. Two people may both be wearing red shirts, but the colors of the shirts may not be exactly the same. Exact descriptions of colors would require reporting the frequency or wavelength of the color.

Quantitative and Qualitative Observations Compared

Table 1.3: **Comparison of Qualitative and Quantitative Observations**

Qualitative (words only)	Quantitative (words and numbers)
The girl has very little money.	The girl has 85 cents.
The man is short.	The man is 5 feet 2 inches tall.
Use a small test tube.	Use a test tube that is 12 centimeters long.
It is a short walk to my house.	It is about 1 mile to my house.

You can see from the last example in **Table** 1.3 that even if the number is an estimate, a quantitative observation contains more information because of the number associated with the observation. Some people might think that a 1-mile walk is short, while others may not. If an actual measuring device is not available, the observer should always try to estimate a measurement so that the observation will have a number associated with it.

While estimated measurements may not be accurate, they are valuable because they establish an approximate numerical description for the observation. "The car is small" is an observation that provides us with certain information. We know that the object is some kind of car (perhaps real or perhaps a toy), and we know that it is probably smaller than a limousine because almost no one would describe a limousine as small. Suppose instead that the observation is: "The car is about 3 feet tall, 3 feet long, and 2 feet wide."

While these estimated measurements are not accurate, we now know that are not dealing with a compact automobile, nor are we dealing with a toy car. With these estimated measurements, we know that we are dealing with a car about the size of a tricycle. It is not a problem if we discover later that the car was actually 2 feet tall instead of 3 feet tall, because we knew the original measurement was an estimate. Estimates are excellent observations if we do not have the ability to measure the object accurately and still qualify as quantitative observations.

Example Questions:

Pick out the quantitative and qualitative observations from each phrase.

1. 3.0 grams of NaCl dissolve in 10 milliliters of H_2O to produce a clear solution.
2. The spider on the wall has only seven legs remaining but is still big and hairy.
3. When 0.50 milliliter of a solution is put into a flame, the flame turns a brilliant green.

Solutions:

1. Quantitative: 3.0 grams and 10 milliliters; Qualitative: clear solution
2. Quantitative: seven legs; Qualitative: big and hairy
3. Quantitative: 0.50 milliliter; Qualitative: brilliant green

Lesson Summary

- Observation involves using the senses to gather information about the natural world.
- There are two types of observations: qualitative and quantitative.
- Scientists gather information by making both qualitative and quantitative observations.
- Qualitative observations yield descriptive, nonnumerical results.
- Quantitative observations yield meaningful, numerical results.
- Observations, either qualitative or quantitative, are used by scientists as tools to make representations and interpretations about the surroundings.

Further Reading / Supplemental Links

This website helps to build your observation skills.

- http://www.mrsoshouse.com/pbl/observe/indexobserve.htm

Review Questions

Label each observation as qualitative or quantitative.

1. The temperature of this room is 25°C.
2. It is comfortably warm in this room.
3. Most people have removed their coats.
4. The building is 25 stories high.
5. It is a very tall building.
6. The building is taller than any nearby trees.
7. The bottle is green.
8. The bottle contains 250 milliliters of liquid.

9. Robert bought his son a small car.
10. The car is smaller than his hand.
11. The car is about three inches long.
12. The race is about 27 miles long.

1.2 Measurement Systems

Lesson Objectives

The student will:

- state an advantage of using the metric system over the United States customary system.
- state the different prefixes used in the metric system.

Vocabulary

- base unit
- metric system

Introduction

Even in ancient times, humans needed measurement systems for commerce. Land ownership required measurements of length, and the sale of food and other commodities required measurements of mass. The first elementary efforts in measurement required convenient objects to be used as standards, such as the human body. Inch and foot are examples of measurement units that are based on parts of the human body. The inch is based on the width of a man's thumb, and the foot speaks for itself. The grain is a unit of mass measurement that is based upon the mass of a single grain of wheat. Because grains of wheat are fairly consistent in mass, the quantity of meat purchased could be balanced against some number of grains of wheat on a merchant's balance.

It should be apparent that measuring the foot of two different people would lead to different results. One way to achieve greater consistency was for everyone to use the foot of one person, such as the king, as the standard. The length of the king's foot could be marked on pieces of wood, and everyone who needed to measure length could have a copy. Of course, this standard would change when a new king was crowned.

What were needed were objects that could be safely stored without changing over time to serve as standards of measurement. Copies of these objects could then be made and distributed so that everyone was using the exact same units of measure. This was especially important when the requirements of science necessitated accurate, reproducible measurements.

The Metric System

The **metric system** is an international decimal-based system of measurement. Because the metric system is a decimal system, making conversions between different units of the metric system are always done with factors of ten. To understand why this makes the metric system so useful and easy to manipulate, let's consider the United States customary system – that is, the measurement system commonly used in the US. For instance, if you need to know how many inches are in a foot, you need to remember: 12 inches = 1 foot. Now imagine that you now need to know how many feet are in a mile. What happens if you have never

memorized this fact before? Of course, you can find this conversion online or elsewhere, but the point is that this information must be given to you, as there is no way for you to derive it by yourself. This is true about all parts of the United States customary system: you have to memorize all the facts that are needed for different measurements.

Metric Prefixes and Equivalents

The metric system uses a number of prefixes along with the base units. A **base unit** is one that cannot be expressed in terms of other units. The base unit of mass is the gram (g), that of length is the meter (m), and that of volume is the liter (L). Each base unit can be combined with different prefixes to define smaller and larger quantities. When the prefix "centi-" is placed in front of gram, as in centigram, the unit is now $\frac{1}{100}$ of a gram. When "milli-" is placed in front of meter, as in millimeter, the unit is now $\frac{1}{1,000}$ of a meter. Common prefixes are shown in **Table** 1.4.

Table 1.4: **Common Prefixes**

Prefix	**Meaning**	**Symbol**
pico-	10^{-12}	p
nano-	10^{-9}	n
micro-	10^{-6}	μ (pronounced *mu*)
milli-	10^{-3}	m
centi-	10^{-2}	c
deci-	10^{-1}	d
kilo-	10^{3}	k

Common metric units, their symbols, and their relationships to a base unit are shown below:

$$\begin{aligned}
1.00 \text{ picogram} &= 1.00 \text{ pg} = 1.00 \times 10^{-12} \text{ g}\\
1.00 \text{ nanosecond} &= 1.00 \text{ ns} = 1.00 \times 10^{-9} \text{ g}\\
1.00 \text{ micrometer} &= 1.00\ \mu\text{m} = 1.00 \times 10^{-6} \text{ m}\\
1.00 \text{ centimeter} &= 1.00 \text{ cm} = 1.00 \times 10^{-2} \text{ m}\\
1.00 \text{ deciliter} &= 1.00 \text{ dL} = 1.00 \times 10^{-1} \text{ L}\\
1.00 \text{ kilogram} &= 1.00 \text{ kg} = 1.00 \times 10^{3} \text{ g}
\end{aligned}$$

You can express a given measurement in more than one unit. If you express a measured quantity in two different metric units, then the two measurements are metric equivalents. Common metric equivalents are shown below.

- Length:

$$\begin{aligned}
1,000 \text{ millimeters} &= 1 \text{ meter}\\
100 \text{ centimeters} &= 1 \text{ meter}\\
10 \text{ millimeters} &= 1 \text{ centimeter}
\end{aligned}$$

- Mass:

$$\begin{aligned}
1,000 \text{ milligrams} &= 1 \text{ gram}\\
1,000 \text{ grams} &= 1 \text{ kilogram}
\end{aligned}$$

- Volume:

1 liter $= 1,000$ milliliters

Lesson Summary

- The metric system is an international decimal-based system of measurement.
- The metric system uses a number of prefixes along with the base units.
- The prefixes in the metric system are multiples of 10.
- A base unit is one that cannot be expressed in terms of other units
- If you express a measured quantity in two different metric units, then the two measurements are metric equivalents.

Further Reading / Supplemental Links

The following website provides more information about the metric system and measurements in chemistry.

- http://www.chemistry24.com/teach_chemistry/measurement-and-math-in-chemistry.html

Review Questions

Fill in the blanks in **Table** 1.5.

Table 1.5: **Table for Review Question**

Prefix	**Meaning**	**Symbol**
pico-	10^{-12}	p
nano-	**?**	n
?	10^{-6}	μ
milli-	10^{-3}	**?**
centi-	**?**	c
deci-	10^{-1}	**?**
?	10^{3}	k

1.3 The SI System of Measurement

Lesson Objectives

The student will:

- explain the difference between mass and weight.
- identify SI units of mass, distance (length), volume, temperature, and time.
- define derived unit.
- describe absolute zero.

Vocabulary

- absolute zero
- cubic meter
- derived units
- heat
- International System of Units
- Kelvin temperature scale
- kilogram
- length
- mass
- meter
- second
- temperature
- volume
- weight

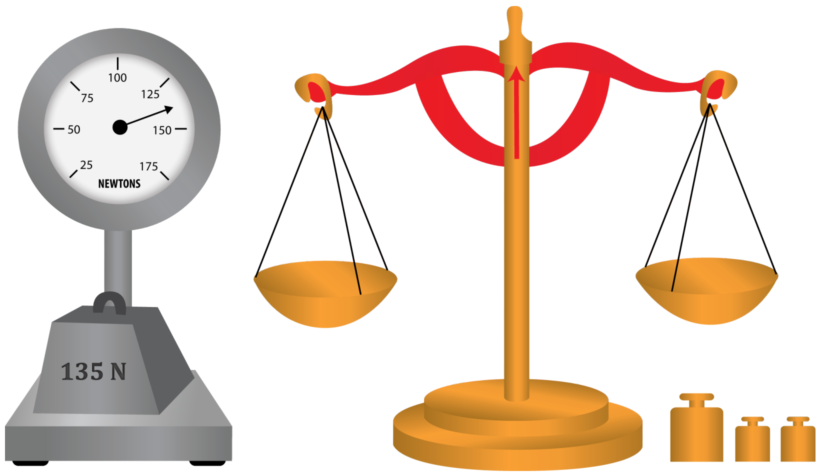
Introduction

The **International System of Units**, abbreviated SI from the French *Le Système International d' Unites*, is the main system of measurement units used in science. Since the 1960s, the International System of Units has been agreed upon internationally as the standard metric system. The SI base units are based on physical standards. The definitions of the SI base units have been and continue to be modified, and new base units are added as advancements in science are made. Each SI base unit, except the kilogram, is described by stable properties of the universe.

Mass and Its SI Unit

Mass and weight are not the same thing. Although we often use these terms interchangeably, each one has a specific definition and usage. The **mass** of an object is a measure of the amount of matter in it and remains the same regardless of where the object is placed. For example, moving a brick to the moon does not cause matter in the brick to disappear or to be removed. The **weight** of an object is the force of attraction between the object and the Earth (or whatever large, gravity-producing body the object is located on). This attraction is due to the force of gravity. Since the force of gravity is not the same at every point on the Earth's surface, the weight of an object is not constant. The gravitational pull on the object varies and depends on where the object is with respect to the Earth or other gravity-producing object. For example, a man who weighs 180 pounds on Earth would weigh only 45 pounds if he were in a stationary position 4,000 miles above the Earth's surface. This same man would weigh only 30 pounds on the moon, because the moon's gravitational pull is one-sixth that of Earth's. The mass of this man, however, would be the same in each situation because the amount of matter in the man is constant.

We measure weight with a scale, which contains a spring that compresses when an object is placed on it. An illustration of a scale is depicted on the left in the diagram below. If the gravitational pull is less, the spring compresses less and the scale shows less weight. We measure mass with a balance, depicted on the right in the diagram below. A balance compares the unknown mass to known masses by balancing them on a lever. If we take our balance and known masses to the moon, an object will have the same measured mass that it had on the Earth. The weight, of course, would be different on the moon. Consistency requires that scientists use mass and not weight when measuring the amount of matter.



The basic unit of mass in the International System of Units is the kilogram. A **kilogram** is equal to 1,000 grams. A gram is a relatively small amount of mass, so larger masses are often expressed in kilograms. When very tiny amounts of matter are measured, we often use milligrams, with one milligram equal to 0.001 gram. Other larger, smaller, or intermediate mass units may also be appropriate.

At the end of the 18th century, a kilogram was the mass of a cubic decimeter of water. In 1889, a new international prototype of the kilogram was made from a platinum-iridium alloy. The kilogram is equal to the mass of this international prototype, which is held in Paris, France. A copy of the standard kilogram is shown in **Figure** 1.1.

Figure 1.1: This image shows a copy of the standard kilogram stored and used in Denmark.

Length and Its SI Unit

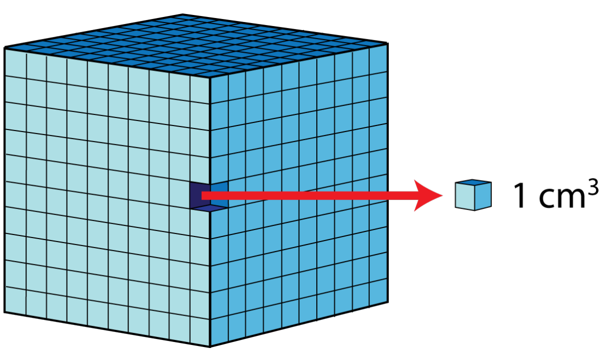
Length is the measurement of anything from end to end. In science, length usually refers to how long an object is. There are many units and sets of standards used in the world for measuring length. The ones familiar to you are probably inches, feet, yards, and miles. Most of the world, however, measure distances in meters and kilometers for longer distances, and in centimeters and millimeters for shorter distances. For consistency and ease of communication, scientists around the world have agreed to use the SI standards, regardless of the length standards used by the general public.

Figure 1.2: This image shows the standard meter used in France in the 18th century.

The SI unit of length is the **meter**. In 1889, the definition of the meter was the length of a bar made of platinum-iridium alloy stored under conditions specified by the International Bureau of Standards. In 1960, this definition of the standard meter was replaced by a definition based on a wavelength of krypton-86 radiation. In 1983, that definition was replaced by the following: the meter is the length of the path traveled by light in a vacuum during a time interval of $\frac{1}{299{,}792{,}458}$ of a second.

Volume: A Derived Unit

The **volume** of an object is the amount of space it takes up. In the International System of Units, volume is a **derived unit**, meaning that it is based on another SI unit. Consider a cube with each side measuring 1.00 meter. The volume of this cube is $1.00\text{ m} \times 1.00\text{ m} \times 1.00\text{ m} = 1.00\text{ m}^3$, or one cubic meter. The **cubic meter** is the SI unit of volume and is based on the meter, the SI unit of length. The cubic meter is a very large unit and is not very convenient for most measurements in chemistry. A more common unit is the liter (L), which is $\frac{1}{1{,}000}$ of a cubic meter. One liter is slightly larger than one quart: 1.000 liter = 1.057 quart. Another commonly used volume measurement is the milliliter, which is equal to $\frac{1}{1{,}000}$ of a liter. Since $\frac{1}{1{,}000}$ of a liter is also equal to 1.00 cubic centimeter, then $1.00\text{ mL} = 1.00\text{ cm}^3$.



As seen in the illustration above, the volume of 1,000 blocks, each with a volume of 1 cubic centimeter, is equivalent to 1 liter.

Measuring Temperature

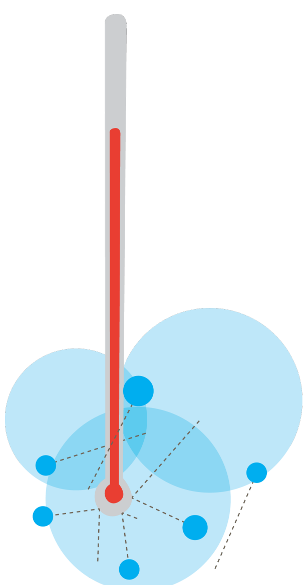
When used in a scientific context, the words heat and temperature do *not* mean the same thing. **Temperature** represents the average kinetic energy of the particles making up a material. Increasing the temperature of a material increases its thermal energy; objects contain thermal energy, not "heat." **Heat**

is the movement of thermal energy from a warmer object to a cooler object. When thermal energy moves from one object to another, the temperature of both objects change. These different types of energies will be re-examined in more detail in later chapters, but the key concept to remember here is that temperature reflects the thermal energy *contained* in an object, while heat is the *movement* of thermal energy between two objects.

Consider a small beaker of boiling water (100°C) and a bathtub of water at a temperature of 50°C. The amount of thermal energy contained in the bathtub is 40,000,000 joules (a measure of energy), while the amount of thermal energy in the beaker is 4,000 joules. Although the temperature of the beaker of water is only twice the temperature of the bathtub of water, the amount of thermal energy contained in the bathtub is many times greater than that in the beaker of water. The important thing to note here is that the amount of thermal energy contained in an object increases greatly with an increase in temperature.

A thermometer is a device that measures temperature. The name is made up of *thermo*, which means heat, and *meter*, which means to measure. One of the earliest inventors of a thermometer was Galileo. He is said to have used a device called a thermoscope around the year 1600. The thermometers we typically use today, however, are different from the one Galileo used.

The type of thermometers most people are familiar with operates on the principle that the volume of most liquids increases when heated and decreases when cooled. If a liquid is trapped inside an evacuated tube with an attached bulb reservoir, like that shown in the diagram below, the liquid in the tube will move higher in the tube when the liquid is heated and lower when the liquid is cooled. After a short period of time, the temperature of the liquid in the bulb will be the same temperature as the surrounding material. The liquid in the tube reflects the temperature of the surrounding because the molecules of material surrounding the bulb will collide with the tube and transfer heat during the process. If heat is transferred to the liquid in the bulb, the liquid will rise and indicate an increase in temperature. If heat is transferred to the surrounding material, the height of the liquid in the tube will fall and indicate a decrease in temperature.



Each thermometer is calibrated by placing it in a liquid whose exact temperature is known. Most thermometers are calibrated using consistent known temperatures that are easy to reproduce. At normal sea level and atmospheric pressure, a stable mixture of ice and water will be at the freezing point of water, and a container of boiling water will be at the boiling point of water. When the height of the liquid inside the thermometer reflects the temperature of the surrounding liquid, a mark (scratch) is made on the tube to indicate that temperature. Once the freezing and boiling temperatures have been marked on

the thermometer, the distance between the marks can be marked up into equal divisions called degrees.

Daniel Fahrenheit established the Fahrenheit scale. On his temperature scale, Fahrenheit designated the freezing point of water as 32°F and the boiling point of water as 212°F. Therefore, the distance between these two points would be divided into 180 degrees. The Fahrenheit temperature scale is used in the United States for most daily expressions of temperature. In another temperature scale established by Anders Celsius, Celsius designated the freezing point of water as 0°C and the boiling point of water as 100°C. Therefore, the temperatures between these two points on the Celsius scale are divided into 100 degrees. Clearly, the size of a Celsius degree and the size of a Fahrenheit degree are not the same.

Earlier in the lesson, the temperature of a substance is defined to be directly proportional to the average kinetic energy it contains. In order for the average kinetic energy and temperature of a substance to be directly proportional, it is necessary for the average kinetic energy to be zero when the temperature is zero. This is not true with either the Fahrenheit or Celsius temperature scales. Most of us are familiar with temperatures that are below the freezing point of water. It should be apparent that even though the air temperature may be −5°C, the molecules of air are still moving. Substances like oxygen and nitrogen have already become vapor at temperatures below −150°C, indicating that molecules are still in motion at over a hundred degrees below zero.

A third temperature scale was established to address this issue. This temperature scale was designed by Lord Kelvin. Lord Kelvin stated that there is no upper limit to how hot things can get, but there is a limit as to how cold things can get. Kelvin developed the idea of **absolute zero**, which is the temperature that molecules stop moving and have zero kinetic energy. The **Kelvin temperature scale** has its zero at absolute zero (determined to be −273.15°C) and uses the same degree size as a degree on the Celsius scale. As a result, the mathematical relationship between the Celsius scale and the Kelvin scale is: $\mathrm{K} = {}^{\circ}\mathrm{C} + 273.15$. On the Kelvin scale, water freezes at 273.15 K and boils at 373.15 K. In the case of the Kelvin scale, the degree sign is not used. Temperatures are expressed, for example, simply as 450 K.

It should be noted that many mathematical calculations in chemistry involve the difference between two temperatures, symbolized by ΔT (pronounced *delta T*). Since the size of a degree is the same in Celsius and in Kelvin, the ΔT will be the same for either scale. For example, 20°C = 293 K and 50°C = 323 K; the difference between the Celsius temperatures is 30°C, and the difference between the Kelvin temperatures is 30 K. When the calculations involve ΔT, it is not necessary to convert Celsius to Kelvin, but when the temperature is used directly in an equation, it *is* necessary to convert Celsius to Kelvin.

This video is an explanation of how to convert among the Celsius, Kelvin, and Fahrenheit temperature scales and includes a sample problem **(4e)**: `http://www.youtube.com/watch?v=SASnMMGp5mo` (4:37).

Figure 1.3: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/353

This video is an explanation of particle temperature, average temperature, heat flow, pressure, and volume **(7a)**: `http://www.youtube.com/watch?v=tfE2y_7LqA4` (4:00).

Figure 1.4: (Watch Youtube Video)
http://www.ck12.org/flexbook/embed/view/354

Time and Its SI Unit

The SI unit for time is the second. The second was originally defined as a tiny fraction of the time required for the Earth to orbit the Sun. It has since been redefined several times. The definition of a **second** (established in 1967 and reaffirmed in 1997) is: the duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

Lesson Summary

- The International System of Units, abbreviated SI from the French *Le Système International d' Unites*, is internationally agreed upon since the 1960s as the standard metric system.
- Mass and weight:
  - The mass of an object is a measure of the amount of matter in it.
  - The mass of an object remains the same regardless of where the object is placed.
  - The basic unit of mass in the International System of Units is the kilogram.
  - The weight of an object is the force of attraction between the object and the earth (or whatever large, gravity-producing body the object is located on).
- Length:
  - Length is the measurement of anything from end to end.
  - The SI unit of length is the meter.
- Volume:
  - The volume of an object is the amount of space it takes up.
  - The cubic meter is the SI unit of volume.
- Temperature and heat:
  - Temperature represents the average kinetic energy of the particles that make up a material.
  - Increasing the temperature of a material increases its thermal energy.
  - Heat is the movement of thermal energy from a warmer object to a cooler object.
  - When thermal energy moves from one object to another, the temperature of both objects change.
  - Absolute zero is the temperature at which molecules stop moving and therefore have zero kinetic energy.
  - The Kelvin temperature scale has its zero at absolute zero (determined to be $-273.15°C$) and uses the same degree size as the Celsius scale.
  - The mathematical relationship between the Celsius scale and the Kelvin scale is $K = °C + 273.15$.
- Time:
  - The SI unit for time is the second.

Review Questions

1. What is the basic unit of measurement in the metric system for length?
2. What is the basic unit of measurement in the metric system for mass?
3. What unit is used in the metric system to measure volume? How is this unit related to the measurement of length?
4. Give the temperatures in Celsius for the freezing and boiling points of water.
5. Give the temperatures in Kelvin for the freezing and boiling points of water.
6. Would it be comfortable to swim in a swimming pool whose water temperature was 275 K? Why or why not?

1.4 Significant Figures

Lesson Objectives

The student will:

- explain the necessity for significant figures.
- determine significant figures of the equipment pieces chosen.
- identify the number of significant figures in a measurement.
- use significant figures properly in measurements and calculations.
- determine the number of significant figures in the result of a calculation.
- round calculated values to the correct number of significant figures.

Vocabulary

- significant figures

Introduction

The numbers you use in math class are considered to be exact numbers. When you are given the number 2 in a math problem, it does not mean 1.999 rounded up to 2, nor does it mean 2.00001 rounded down to 2. In math class, the number 2 means exactly 2.000000... with an infinite number of zeros – a perfect 2! Such numbers are produced only by definition, *not* by measurement. We can define 1 foot to contain exactly 12 inches with both numbers being perfect numbers, but we cannot measure an object to be exactly 12 inches long. In the case of measurements, we can only read our measuring instruments to a limited number of subdivisions. We are limited by our ability to see smaller and smaller subdivisions, and we are limited by our ability to construct smaller and smaller subdivisions on our measuring devices. Even with the use of powerful microscopes to construct and read our measuring devices, we eventually reach a limit. Therefore, although the actual measurement of an object may be a perfect 12 inches, we cannot prove it to be so. Measurements do not produce perfect numbers; the only perfect numbers in science are defined numbers, such as conversion factors. Since measurements are fundamental to science, science does not produce perfect measurements.

It is very important to recognize and report the limitations of a measurement along with the magnitude and unit of the measurement. Many times, the measurements made in an experiment are analyzed for regularities. If the numbers reported show the limits of the measurements, the regularity, or lack thereof, becomes visible.

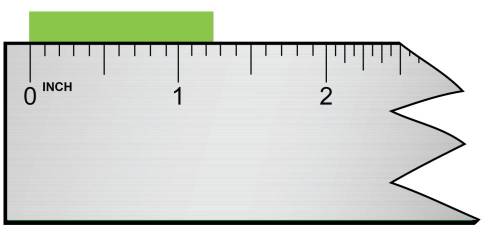
Table 1.6: **Comparison of Observations with the Proper Number of Significant Figures**

Observation List A	Observation List B
22.41359 m	22.4 m
22.37899 m	22.4 m
22.42333 m	22.4 m
22.39414 m	22.4 m

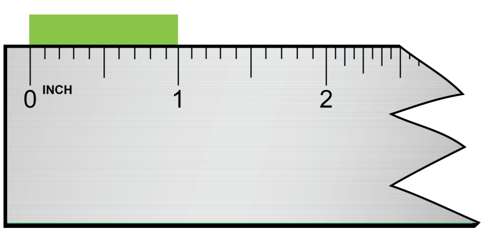
In the lists of observations shown in **Table** 1.6, List A shows measurements without including the limits of the measuring device. In comparison, List B has the measurements rounded to reflect the limits of the measuring device. It is difficult to perceive regularity in List A, but the regularity stands out in List B.

Rules for Determining Significant Figures

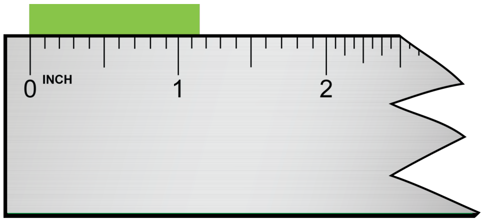
Significant figures, also known as significant digits, are all of the digits that can be known with certainty in a measurement plus an estimated last digit. Significant figures provide a system to keep track of the limits of the original measurement. To record a measurement, you must write down all the digits actually measured, including measurements of zero, and you must *not* write down any digit not measured. The only real difficulty with this system is that zeros are sometimes used as measured digits, while other times they are used to locate the decimal point.



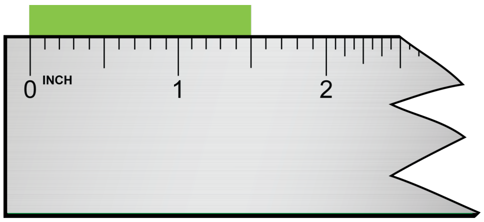
In the sketch shown above, the correct measurement is greater than 1.2 inches but less than 1.3 inches. It is proper to estimate one place beyond the calibrations of the measuring instrument. This ruler is calibrated to 0.1 inches, so we can estimate the hundredths place. This reading should be reported as 1.25 or 1.26 inches.



In this second case (sketch above), it is apparent that the object is, as nearly as we can read, 1 inch. Since we know the tenths place is zero and can estimate the hundredths place to be zero, the measurement should be reported as 1.00 inch. It is vital that you include the zeros in your reported measurement because these are measured places and are significant figures.



This measurement is read as 1.15 inches, 1.16 inches, or perhaps even 1.17 inches.



This measurement is read as 1.50 inches.

In all of these examples, the measurements indicate that the measuring instrument had subdivisions of a tenth of an inch and that the hundredths place is estimated. There is some uncertainty about the last, and only the last, digit.

In our system of writing measurements to show significant figures, we must distinguish between measured zeros and place-holding zeros. Here are the rules for determining the number of significant figures in a measurement.

Rules for Determining the Number of Significant Figures:

1. All non-zero digits are significant.
2. All zeros between non-zero digits are significant.
3. All beginning zeros are *not* significant.
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal (the decimal point is not written in).

Examples of the Significant Figure Rules:

1. All non-zero digits are significant.

543	has 3 significant figures.
22.437	has 5 significant figures.
1.321754	has 7 significant figures.

2. All zeros between non-zero digits are significant.

$7,004$	has 4 significant figures.
10.3002	has 6 significant figures.
103	has 3 significant figures.

3. All beginning zeros are *not* significant.

0.00000075	has 2 significant figures.
0.02	has 1 significant figure.
0.003003	has 4 significant figures.

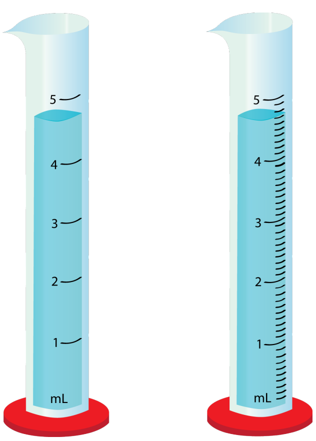
4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.

37.300	has 5 significant figures.
33.00000	has 7 significant figures.
100.	has 3 significant figures.
100	has 1 significant figure.
302, 000	has 3 significant figures.
1, 050	has 3 significant figures.

Equipment Determines Significant Figures

Quality measuring instruments are made with as much consistency as possible and are individually calibrated after construction. In a graduated cylinder, for example, it is desirable for the sides to be perfectly vertical and for the inside diameter to be the same all the way up the tube. After the graduated cylinder is completed, exact volumes of liquids are placed in the cylinder and the calibration marks are then scribed onto the side of the tube.

The choice of measuring instrument determines the unit of measure and the number of significant figures in the measurement. Consider the two graduated cylinders shown below.



Both cylinders are marked to measure milliliters, but the cylinder on the left only shows graduations for whole milliliters. In comparison, the cylinder on the right has calibrations for tenths of milliliters. The measurer reads the volume from the calibrations and estimates one place beyond the calibrations. For the cylinder on the left, a reasonable reading is 4.5 mL. For the cylinder on the right, the measurer estimates one place beyond the graduations and obtains a reasonable reading of 4.65 mL. The choice of the measuring instrument determines both the units and the number of significant figures. If you were mixing up some hot chocolate at home, the cylinder on the left would be adequate. If you were measuring out a chemical solution for a very delicate reaction in the lab, however, you would need the cylinder on the right.

Similarly, the equipment chosen for measuring mass will also affect the number of significant figures. For example, if you use a pan balance (illustrated on the left in the image below) that can only measure to ± 0.1 g, you could only measure out 3.3 g of NaCl rather than 3.25 g. In comparison, the digital balance (illustrated on the right in the image below) might be able to measure to ± 0.01 g. With this instrument, you could measure what you need more exactly. The difference between these two balances has to do with the number of significant figures that the balances are able to measure. Whenever you need to make a measurement, make sure to check the number of significant figures a measuring instrument can measure before choosing an appropriate instrument.

Significant Figures in Calculations

In addition to using significant figures to report measurements, we also use them to report the results of computations made with measurements. The results of mathematical operations on measurements must indicate the number of significant figures in the original measurements. There are two rules for determining the number of significant figures after performing a mathematical operation. Most of the errors that occur in this area result from using the wrong rule, so always double check that you are using the correct rule for the mathematical operation involved.

Addition and Subtraction

The answer to an addition or subtraction operation must not have any digits further to the right than the shortest addend. In other words, the answer should have as many decimal places as the addend with the smallest number of decimal places.

Example:

```
 13.3843 cm
  1.012  cm
+ 3.22   cm
 ----------
 17.6163 cm = 17.62 cm
```

Notice that the top addend has a 3 in the last column on the right, but neither of the other two addends have a number in that column. In elementary math classes, you were taught that these blank spaces can be filled in with zeros and the answer would be 17.6163 cm. In the sciences, however, these blank spaces are unknown numbers, *not* zeros. Since they are unknown numbers, you cannot substitute any numbers into the blank spaces. As a result, you cannot know the sum of adding (or subtracting) any column of numbers that contain an unknown number. When you add the columns of numbers in the example above, you can only be certain of the sums for the columns with known numbers in each space in the column. In science, the process is to add the numbers in the normal mathematical process and then round off all columns that contain an unknown number (a blank space). Therefore, the correct answer for the example above is 17.62 cm and has only four significant figures.

Example:

```
 12        m
+ 0.00045 m
 ----------
 12.00045 m = 12 m
```

In this case, the addend 12 has no digits beyond the decimal. Therefore, all columns past the decimal point must be rounded off in the final answer. We get the seemingly odd result that the answer is still 12,

even after adding a number to 12. This is a common occurrence in science and is absolutely correct.

Example:

```
  56.8885 cm
   8.30   cm
+ 47.0    cm
 112.1885 cm = 112.2 cm
```

Multiplication and Division

The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

Example:

$$(3.556 \text{ cm}) \cdot (2.4 \text{ cm}) = 8.5344 \text{ cm}^2 = 8.5 \text{ cm}^2$$

The factor 3.556 cm has four significant figures, and the factor 2.4 cm has two significant figures. Therefore the answer must have two significant figures. The mathematical answer of 8.5344 cm^2 must be rounded back to 8.5 cm^2 in order for the answer to have two significant figures.

Example:

$$(20.0 \text{ cm}) \cdot (5.0000 \text{ cm}) = 100.00000 \text{ cm}^2 = 100. \text{ cm}^2$$

The factor 20.0 cm has three significant figures, and the factor 5.0000 cm has five significant figures. The answer must be rounded to three significant figures. Therefore, the decimal must be written in to show that the two ending zeros are significant. If the decimal is omitted (left as an understood decimal), the two zeros will not be significant and the answer will be wrong.

Example:

$$(5.444 \text{ cm}) \cdot (22 \text{ cm}) = 119.768 \text{ cm}^2 = 120 \text{ cm}^2$$

In this case, the answer must be rounded back to two significant figures. We cannot have a decimal after the zero in 120 cm^2 because that would indicate the zero is significant, whereas this answer must have exactly two significant figures.

Lesson Summary

- Significant figures are all of the digits that can be known with certainty in a measurement plus an estimated last digit.
- Significant figures provide a system to keep track of the limits of a measurement.
- Rules for determining the number of significant figures:
  1. All non-zero digits are significant.
  2. All zeros between non-zero digits are significant.
  3. All beginning zeros are *not* significant.
  4. Ending zeros are significant if the decimal point is actually written in but *not* significant if the decimal point is an understood decimal.

- The choice of measuring instrument is what determines the unit of measure and the number of significant figures in the measurement.
- The results of mathematical operations must include an indication of the number of significant figures in the original measurements.
- The answer for an addition or subtraction operation must not have any digits further to the right than the shortest addend.
- The answer for a multiplication or division operation must have the same number of significant figures as the factor with the least number of significant figures.

Further Reading / Supplemental Links

A problem set on unit conversions and significant figures.

- http://science.widener.edu/svb/pset/convert1.html

Review Questions

1. How many significant figures are in the following numbers?
   (a) 2.3
   (b) 17.95
   (c) 9.89×10^3
   (d) 170
   (e) 22.1
   (f) 1.02
   (g) 19.84
2. Perform the following calculations and give your answer with the correct number of significant figures:
   (a) $10.5 + 11.62$
   (b) $0.01223 + 1.01$
   (c) $19.85 - 0.0113$
3. Perform the following calculations and give your answer with the correct number of significant figures:
   (a) 0.1886×12
   (b) $2.995 \div 0.16685$
   (c) $1210 \div 0.1223$
   (d) 910×0.18945

1.5 Using Algebra in Chemistry

Lesson Objectives

The student will:

- perform algebraic manipulations to solve equations.
- use the density equation to solve for the density, mass, or volume when two of the quantities in the equation are known.
- construct conversion factors from equivalent measurements.
- apply the techniques of dimensional analysis to solving problems.
- perform metric conversions using dimensional analysis.

Vocabulary

- conversion factor
- dimensional analysis

Introduction

During your studies of chemistry (and physics as well), you will note that mathematical equations are used in a number of different applications. Many of these equations have a number of different variables that you will need to work with. You should also note that these equations will often require you to use measurements with their units. Algebra skills become very important here!

Solving Algebraic Equations

Chemists use algebraic equations to express relationships between quantities. An example of such an equation is the relationship between the density, mass, and volume of a substance: $D = \frac{m}{V}$ (density is equal to mass divided by volume). Given (or making) measurements of the mass and volume of a substance, you can use this equation to determine the density. Suppose, for example, that you have measured the mass and volume of a sample of liquid mercury and found that 5.00 mL of mercury has a mass of 67.5 grams. Plugging these measurements into the density formula gives you the density of mercury.

$$D = \frac{\text{mass}}{\text{volume}} = \frac{67.5 \text{ g}}{5.00 \text{ mL}} = 13.5 \text{ g/mL}$$

You should notice both units and significant figures are carried through the mathematical operations.

Frequently, you may be asked to use the density equation to solve for a variable other than density. For example, you may be given measurements for density and mass and be asked to determine the volume.

Example:

The density of solid lead is 11.34 g/mL. What volume will 81.0 g of lead occupy?

Since $D = \frac{m}{V}$, then $V = \frac{m}{D}$.
$$V = \frac{81.0 \text{ g}}{11.34 \text{ g/mL}} = 7.14 \text{ mL}$$

A common equation used in chemistry is $PV = nRT$. Even without knowing what these variables represent, you can manipulate this equation to solve for any of the five quantities.

$$P = \frac{nRT}{V} \qquad V = \frac{nRT}{P} \qquad n = \frac{PV}{RT} \qquad R = \frac{PV}{nT} \qquad T = \frac{PV}{nR}$$

Make sure you recall these skills from algebra. If necessary, you should practice them.

Example:

Use the equation $\frac{A}{B} = \frac{C}{D}$ and the values $A = 15.1$ g, $B = 3.000$ mL, and $C = 326.96$ grams to determine the value of D.

$$D = \frac{BC}{A} = \frac{(3.000 \text{ mL})(326.96 \text{ g})}{(15.1 \text{ g})} = 65.0 \text{ mL}$$

The calculator-determined value for this arithmetic may yield 64.956954 mL but you now know not to report such a value. Since this answer only allows three significant figures, you must round the answer to 65.0 mL.

Conversion Factors

A **conversion factor** is a factor used to convert one unit of measurement into another unit. A simple conversion factor can be used to convert meters into centimeters, or a more complex one can be used to convert miles per hour into meters per second. Since most calculations require measurements to be in certain units, you will find many uses for conversion factors. What must always be remembered is that a conversion factor has to represent a fact; because the conversion factor is a fact and not a measurement, the numbers in a conversion factor are exact. This fact can either be simple or complex. For instance, you probably already know the fact that 12 eggs equal 1 dozen. A more complex fact is that the speed of light is 1.86×10^5 miles/second. Either one of these can be used as a conversion factor, depending on the type of calculation you might be working with.

Dimensional Analysis

Frequently, it is necessary to convert units measuring the same quantity from one form to another. For example, it may be necessary to convert a length measurement in meters to millimeters. This process is quite simple if you follow a standard procedure called dimensional analysis (also known as unit analysis or the factor-label method). **Dimensional analysis** is a technique that involves the study of the dimensions (units) of physical quantities. It is a convenient way to check mathematical equations. Dimensional analysis involves considering the units you presently have and the units you wish to end up with, as well as designing conversion factors that will cancel units you don't want and produce units you do want. The conversion factors are created from the equivalency relationships between the units. For example, one unit of work is a newton meter (abbreviated $\text{N} \cdot \text{m}$). If you have measurements in newtons (a unit for force, F) and meters (a unit for distance, d), how would you calculate work? An analysis of the units will tell you that you should multiply force times distance to get work: $W = F \times d$.

Suppose you want to convert 0.0856 meters into millimeters. In this case, you need only one conversion factor that will cancel the meters unit and create the millimeters unit. The conversion factor will be created from the relationship 1000 mL = 1 m.

$$(0.0856 \text{ m}) \cdot (\tfrac{1000 \text{ mm}}{1 \text{ m}}) = (0.0856 \cancel{\text{m}}) \cdot (\tfrac{1000 \text{ mm}}{1 \cancel{\text{m}}}) = 85.6 \text{ mm}$$

In the above expression, the meter units will cancel and only the millimeter unit will remain.

Example:

Convert 1.53 g to cg.

The equivalency relationship is 1.00 g = 100 cg, so the conversion factor is constructed from this equivalency in order to cancel grams and produce centigrams.

$$(1.53 \text{ g}) \cdot (\tfrac{100 \text{ cg}}{1 \text{ g}}) = 153 \text{ cg}$$

Example:

Convert 1000. in. to ft.

The equivalency between inches and feet is 12 in. = 1 ft. The conversion factor is designed to cancel inches and produce feet.

$$(1000. \text{ in.}) \cdot (\tfrac{1 \text{ ft}}{12 \text{ in.}}) = 83.33 \text{ ft}$$

Each conversion factor is designed specifically for the problem. In the case of the conversion above, we need to cancel inches, so we know that the inches component in the conversion factor needs to be in the denominator.

Example:

Convert 425 *klums* to *piks* given the equivalency relationship 10 *klums* = 1 *pik*.

$$(425\ klums) \cdot (\frac{1\ pik}{10\ klums}) = 42.5\ piks$$

Sometimes, it is necessary to insert a series of conversion factors. Suppose we need to convert miles to kilometers, and the only equivalencies we know are 1 mi = 5,280 ft, 12 in. = 1 ft, 2.54 cm = 1 in., 100 cm = 1 m, and 1000 m = 1 km. We will set up a series of conversion factors so that each conversion factor produces the next unit in the sequence.

Example:

Convert 12 mi to km.

$$(12\ \text{mi}) \cdot (\frac{5280\ \ \text{ft}}{1\ \text{mi}}) \cdot (\frac{12\ \text{in.}}{1\ \text{ft}}) \cdot (\frac{2.54\ \text{cm}}{1\ \text{in.}}) \cdot (\frac{1\ \text{m}}{100\ \text{cm}}) \cdot (\frac{1\ \text{km}}{1000\ \text{m}}) = 19\ \text{km}$$

In each step, the previous unit is canceled and the next unit in the sequence is produced.

Conversion factors for area and volume can also be produced by this method.

Example:

Convert 1500 cm^2 to m^2.

$$(1500\ \text{cm}^2) \cdot (\frac{1\ \text{m}}{100\ \text{cm}})^2 = (1500\ \text{cm}^2) \cdot (\frac{1\ \text{m}^2}{10{,}000\ \text{cm}^2}) = 0.15\ \text{m}^2$$

Example:

Convert 12.0 in^3 to cm^3.

$$(12.0\ \text{in}^3) \cdot (\frac{2.54\ \text{cm}}{1\ \text{in}})^3 = (12.0\ \text{in}^3) \cdot (\frac{16.4\ \text{cm}^3}{1\ \text{in}^3}) = 197\ \text{cm}^3$$

Lesson Summary

- Conversion factors are used to convert one unit of measurement into another unit.
- Dimensional analysis involves considering both the units you presently have and the units you wish to end up with, as well as designing conversion factors that will cancel units you don't want and produce units you do want.

Further Reading / Supplemental Links

Visit this website for a video series that reviews topics on measurement.

- http://www.learner.org/resources/series184.html

This website provides a math skills review on dimensional analysis.

- http://www.chem.tamu.edu/class/fyp/mathrev/mr-da.html

Review Questions

1. For the equation $PV = nRT$, re-write it so that it is in the form of $T =$.
2. The equation for density is $D = \frac{m}{V}$. If D is 12.8 g/cm^3 and m is 46.1 g, solve for V, keeping significant figures in mind.
3. The equation $P_1 \cdot V_1 = P_2 \cdot V_2$, known as Boyle's law, shows that gas pressure is inversely proportional to its volume. Re-write Boyle's law so it is in the form of $V_1 =$.
4. The density of a certain solid is measured and found to be 12.68 g/mL. Convert this measurement into kg/L.
5. In a nuclear chemistry experiment, an alpha particle is found to have a velocity of $14,285$ m/s. Convert this measurement into miles/hour (mi/h).

1.6 Scientific Notation

Lesson Objectives

The student will:

- use scientific notation to express large and small numbers.
- add, subtract, multiply, and divide using scientific notation.

Vocabulary

- scientific notation

Introduction

Work in science frequently involves very large and very small numbers. The speed of light, for example, is 300,000,000 m/s; the mass of the earth is 6,000,000,000,000,000,000,000,000 kg; and the mass of an electron is 0.0000000000000000000000000000009 kg. It is very inconvenient to write out such numbers and even more inconvenient to attempt to carry out mathematical operations with them. Scientists and mathematicians have designed an easier method to deal with such long numbers. This more convenient system is called exponential notation by mathematicians and scientific notation by scientists.

What is Scientific Notation?

In **scientific notation**, very large and very small numbers are expressed as the product of a number between 1 and 10 multiplied by some power of 10. For example, the number $9,000,000$ can be written as the product of 9 times $1,000,000$. In turn, $1,000,000$ can be written as 10^6. Therefore, $9,000,000$ can be written as 9×10^6. In a similar manner, 0.00000004 can be written as 4 times $\frac{1}{10^8}$, or 4×10^{-8}.

Table 1.7: **Examples of Scientific Notation**

Decimal Notation	Scientific Notation
$95,672$	9.5672×10^4
$8,340$	8.34×10^3
100	1×10^2
7.21	7.21×10^0
0.014	1.4×10^{-2}
0.0000000080	8.0×10^{-9}
0.00000000000975	9.75×10^{-12}

As you can see from the examples in **Table** 1.7, to convert a number from decimal form into scientific notation, you count the number of spaces needed to move the decimal, and that number becomes the exponent of 10. If you are moving the decimal to the left, the exponent is positive, and if you are moving the decimal to the right, the exponent is negative. You should note that *all significant figures are maintained in scientific notation.* You will probably realize that the greatest advantage of using scientific notation occurs when there are many non-significant figures.

Scientific Notation in Calculations

Addition and Subtraction

When numbers in exponential form are added or subtracted, the exponents must be the same. If the exponents are the same, the coefficients are added and the exponent remains the same.

Example:

$$(4.3 \times 10^4) + (1.5 \times 10^4) = (4.3 + 1.5) \times 10^4 = 5.8 \times 10^4$$

Note that the example above is the same as:

$$43,000 + 15,000 = 58,000 = 5.8 \times 10^4.$$

Example:

$$(8.6 \times 10^7) - (5.3 \times 10^7) = (8.6 - 5.3) \times 10^7 = 3.3 \times 10^7$$

Example:

$$(8.6 \times 10^5) + (3.0 \times 10^4) = ?$$

These two exponential numbers do not have the same exponent. If the exponents of the numbers to be added or subtracted are not the same, then one of the numbers must be changed so that the two numbers have the same exponent. In order to add them, we can change the number 3.0×10^4 to 0.30×10^5. This change is made by moving the decimal one place to the left and increasing the exponent by one. Now the two numbers can be added.

$$(8.6 \times 10^5) + (0.30 \times 10^5) = (8.6 + 0.30) \times 10^5 = 8.9 \times 10^5$$

We could also have chosen to alter the other number. Instead of changing the second number to a higher exponent, we could have changed the first number to a lower exponent.

8.6×10^5 becomes 86×10^4
$(86 \times 10^4) + (3.0 \times 10^4) = (86 + 3.0) \times 10^4 = 89 \times 10^4$

Even though it is not always necessary, the preferred practice is to express exponential numbers in proper form, which has only one digit to the left of the decimal. When 89×10^4 is converted to proper form, it becomes 8.9×10^5, which is precisely the same result as before.

Multiplication and Division

When multiplying or dividing numbers in exponential form, the numbers do not have to have the same exponents. To multiply exponential numbers, multiply the coefficients and add the exponents. To divide exponential numbers, divide the coefficients and subtract the exponents.

Multiplication Examples:

$(4.2 \times 10^4) \cdot (2.2 \times 10^2) = (4.2 \cdot 2.2) \times 10^{4+2} = 9.2 \times 10^6$

The product of 4.2 and 2.2 is 9.24, but since we are limited to two significant figures, the coefficient is rounded to 9.2.

$(2 \times 10^9) \cdot (4 \times 10^{14}) = (2 \cdot 4) \times 10^{9+14} = 8 \times 10^{23}$
$(2 \times 10^{-9}) \cdot (4 \times 10^4) = (2 \cdot 4) \times 10^{-9+4} = 8 \times 10^{-5}$
$(2 \times 10^{-5}) \cdot (4 \times 10^{-4}) = (2 \cdot 4) \times 10^{(-5)+(-4)} = 8 \times 10^{-9}$
$(8.2 \times 10^{-9}) \cdot (8.2 \times 10^{-4}) = (8.2 \cdot 8.2) \times 10^{(-9)+(-4)} = 67.24 \times 10^{-13}$

In this last example, the product has too many significant figures and is not in proper exponential form. We must round to two significant figures and adjust the decimal and exponent. The correct answer would be 6.7×10^{-12}.

Division Examples:

$\frac{8\times10^7}{2\times10^4} = 4 \times 10^{7-4} = 4 \times 10^3$
$\frac{8\times10^{-7}}{2\times10^{-4}} = 4 \times 10^{(-7)-(-4)} = 4 \times 10^{-3}$
$\frac{4.6\times10^3}{2.3\times10^{-4}} = 2.0 \times 10^{(3)-(-4)} = 2.0 \times 10^7$

In the example above, since the original coefficients have two significant figures, the answer must also have two significant figures. Therefore, the zero in the tenths place is written to indicate the answer has two significant figures.

Lesson Summary

- Very large and very small numbers in science are expressed in scientific notation.
- All significant figures are maintained in scientific notation.
- When numbers in exponential form are added or subtracted, the exponents must be the same. If the exponents are the same, the coefficients are added and the exponent remains the same.
- To multiply exponential numbers, multiply the coefficients and add the exponents.
- To divide exponential numbers, divide the coefficients and subtract the exponents.

Review Questions

1. Write the following numbers in scientific notation.
   (a) 0.0000479
   (b) $251,000,000$
   (c) $4,260$
   (d) 0.00206

Do the following calculations without a calculator.

2. $(2.0 \times 10^3) \cdot (3.0 \times 10^4)$
3. $(5.0 \times 10^{-5}) \cdot (5.0 \times 10^8)$
4. $(6.0 \times 10^{-1}) \cdot (7.0 \times 10^{-4})$
5. $\frac{(3.0 \times 10^{-4}) \cdot (2.0 \times 10^{-4})}{2.0 \times 10^{-6}}$

Do the following calculations.

6. $(6.0 \times 10^7) \cdot (2.5 \times 10^4)$
7. $\frac{4.2 \times 10^{-4}}{3.0 \times 10^{-2}}$

1.7 Evaluating Measurements

Lesson Objectives

The student will:

- define accuracy and precision.
- explain the difference between accuracy and precision.
- indicate whether a given data set is precise, accurate, both, or neither.
- calculate percent error in an experiment.

Vocabulary

- accuracy
- percent error
- precision

Introduction

Accuracy and precision are two words that we hear a lot in science, math, and other everyday events. They are also, surprisingly, two words that are often misused. For example, you may hear car advertisements talking about the car's ability to handle precision driving. But what do these two words mean?

Accuracy and Precision

Every measurement compares the physical quantity being measured with a fixed standard of measurement, such as the centimeter or the gram. In describing the reliability of a measurement, scientists often use the terms accuracy and precision. **Accuracy** refers to how close a measurement is to the true value of the quantity being measured. **Precision** refers to how close the values in a set of measurements are to one another. If you are using a flawed measuring instrument, you could get very precise measurements (meaning they are very reproducible), but the measurements would be inaccurate. In many cases, the true value of the measurement is not known, and we must take our measurement as the true value. In such cases, instruments are checked carefully to verify that they are unflawed before a series of precise measurements are made. It is assumed that good instruments and precise measurements imply accuracy.

Suppose a student made the same volume measurement four times and obtained the following measurements: 34.25 mL, 34.45 mL, 34.33 mL, and 34.20 mL. The average of these four readings is 34.31 mL. If the actual volume was known to be 34.30 mL, what could we say about the accuracy and precision of these measurements, and how much confidence would we have in the answer? Since the final average is very close to the actual value, we would say that the answer is accurate. However, the individual readings are not close to each other, so we would conclude that the measurements were not precise. If we did not know the correct answer, we would have very little confidence that these measurements produced an accurate value.

Consider the values obtained by another student making the same measurements: 35.27 mL, 35.26 mL, 35.27 mL, and 35.28 mL. In this case, the average measurement is 35.27 mL, and the set of measurements is quite precise since all readings are within 0.1 mL of the average measurement. We would normally have confidence in this measurement since the precision is so good, but if the actual volume is 34.30 mL, the measurements are not accurate. Generally, situations where the measurements are precise but not accurate are caused by a flawed measuring instrument. The ideal situation is to have quality measuring instruments so that precision will imply accuracy.

Percent Error

Percent error is a common way of evaluating the accuracy of a measured value. Anytime an experiment is conducted, a certain degree of uncertainty and error is expected. Scientists often express this uncertainty and error in measurement by reporting a percent error.

$$\text{percent error} = \frac{\text{(accepted value - experimental value)}}{\text{(accepted value)}} \times 100\%$$

The experimental value is what you recorded or calculated based on your own experiment in the lab. The value that can be found in reference tables is called the accepted value. Percent error is a measure of how far the experimental value is from the accepted value.

Example:

A student determined the density of a sample of silver to be 10.3 g/cm^3. The density of silver is actually 10.5 g/cm^3. What is the percent error in the experimentally determined density of silver?

$$\text{percent error} = \frac{10.5\ \text{g/cm}^3 - 10.3\ \text{g/cm}^3}{10.5\ \text{g/cm}^3} \times 100\% = 1.90\%$$

Lesson Summary

- Accuracy reflects how close the measured value is to the actual value.

- Precision reflects how close the values in a set of measurements are to each other.
- Accuracy is affected by the quality of the instrument or measurement.
- Percent error is a common way of evaluating the accuracy of a measured value.
- percent error $= \frac{\text{(accepted value - experimental value)}}{\text{(accepted value)}} \times 100\%$

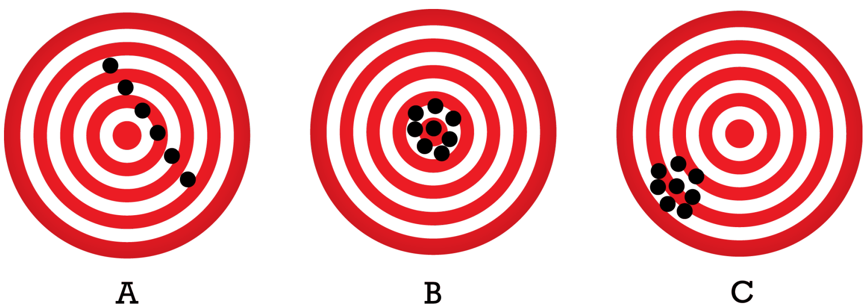
Further Reading / Supplemental Links

The *learner.org* website allows users to view streaming videos of the Annenberg series of chemistry videos. You are required to register before you can watch the videos, but there is no charge to register. The website has a video that apply to this lesson called "Measurement: The Foundation of Chemistry" that details the value of accuracy and precision.

- http://learner.org/resources/series61.html

Review Questions

1. Suppose you want to hit the center of this circle with a paint ball gun. Which of the following are considered accurate? Precise? Both? Neither?



2. Four students take measurements to determine the volume of a cube. Their results are 15.32 cm^3, 15.33 cm^3, 15.33 cm^3, and 15.31 cm^3. The actual volume of the cube is 16.12 cm^3. What statement(s) can you make about the accuracy and precision in their measurements?
3. Distinguish between accuracy and precision.
4. Nisi was asked the following question on her lab exam: When doing an experiment, what term best describes the reproducibility in your results? What should she answer?
   (a) accuracy
   (b) care
   (c) precision
   (d) significance
   (e) uncertainty
5. Karen was working in the lab doing reactions involving mass. She needed to weigh out 1.50 g of each reactant and put them together in her flask. She recorded her data in her data table (**Table** 1.8). What can you conclude by looking at Karen's data?
   (a) The data is accurate but not precise.
   (b) The data is precise but not accurate.
   (c) The data is neither precise nor accurate.
   (d) The data is precise and accurate.
   (e) You really need to see the balance Karen used.

Table 1.8: **Data Table for Problem 5**

	Mass of Reactant A	Mass of Reactant B
Trial 1	1.47 ± 0.02 g	1.48 ± 0.02 g
Trial 2	1.46 ± 0.02 g	1.46 ± 0.02 g
Trial 3	1.48 ± 0.02 g	1.50 ± 0.02 g

6. John uses his thermometer and finds the boiling point of ethanol to be 75°C. He looks in a reference book and finds that the actual boiling point of ethanol is 78°C. What is his percent error?
7. The density of water at 4°C is known to be 1.00 g/mL. Kim experimentally found the density of water to be 1.085 g/mL. What is her percent error?
8. An object has a mass of 35.0 g. On a digital balance, Huey finds the mass of the object to be 34.92 g. What is the percent error of his balance?

1.8 Graphing

Lesson Objectives

The student will:

- correctly graph data with the proper scale, units, and best fit curve.
- recognize patterns in data from a graph.
- solve for the slope of given line graphs.

Vocabulary

- extrapolation
- graph
- interpolation
- slope

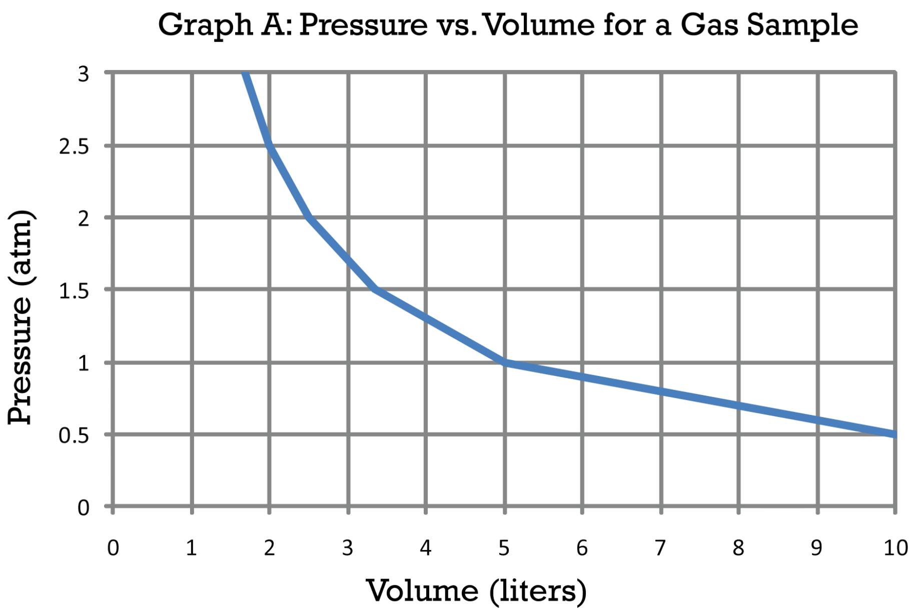
Introduction

Scientists search for regularities and trends in data. To make it easier to find these regularities and trends, scientists often present data in either a table or a graph. The table below presents data about the pressure and volume of a sample of gas. You should note that all tables have a title and include the units of the measurements. The unit of pressure used here is atm (atmosphere).

Data Table A: Pressure vs. Volume Data for a Gas Sample

Pressure (in atm)	**Volume (in liters)**
0.50	10.0
1.00	5.00
1.50	3.33
2.00	2.50
2.50	2.00
3.00	1.67

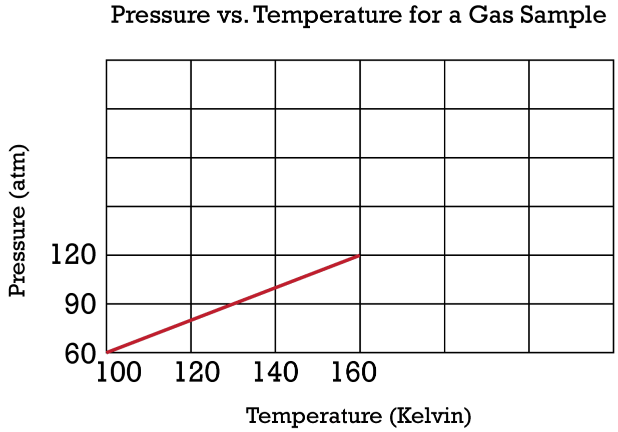
You may note a regularity that appears in this table: as the pressure of the gas increases, its volume decreases. This regularity or trend becomes even more apparent in a graph of this data. A **graph** is a pictorial representation of the relationship between variables on a coordinate system.

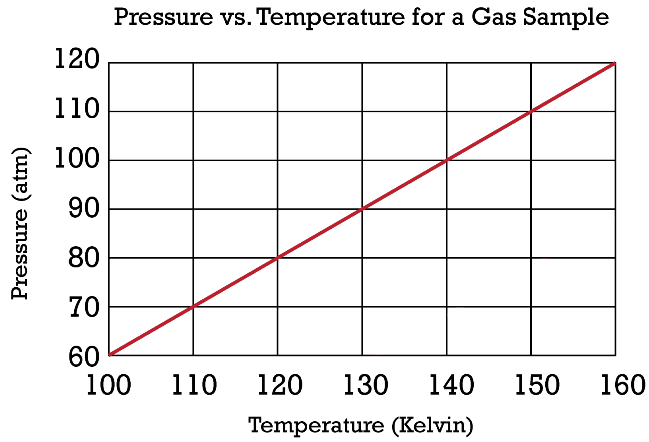


When the data from Data Table A is plotted as a graph, the trend in the relationship between the pressure and volume of a gas sample becomes more apparent. The graph aids the scientist in the search for any regularity that may exist in the data.

Drawing Line Graphs

Reading information from a line graph is easier and more accurate as the size of the graph increases. In the example below, the graph on the left uses only a small fraction of the space available on the graph paper. The graph on the right shows the same data but uses all the space available. If you were attempting to determine the pressure at a temperature of 110 K, using the graph on the left would give a less accurate result than using the graph on the right.





When you draw a line graph, you should arrange the numbers on the axes to use as much of the graph paper as you can. If the lowest temperature in your data is 100 K and the highest temperature in your data is 160 K, you should arrange for 100 K to be on the extreme left of your graph and 160 K to be on the extreme right of your graph. The creator of the graph on the left did not take this advice and did not produce a very good graph. You should also make sure that the axes on your graph are labeled and that your graph has a title.

Reading Information from a Graph

When we draw a line graph from a set of data points, we are inferring a trend and constructing new data points between known data points. This process is called **interpolation**. Even though we may only have a few data points, we are estimating the values between measured points, assuming that the line connecting these data points is a good model of what we're studying.

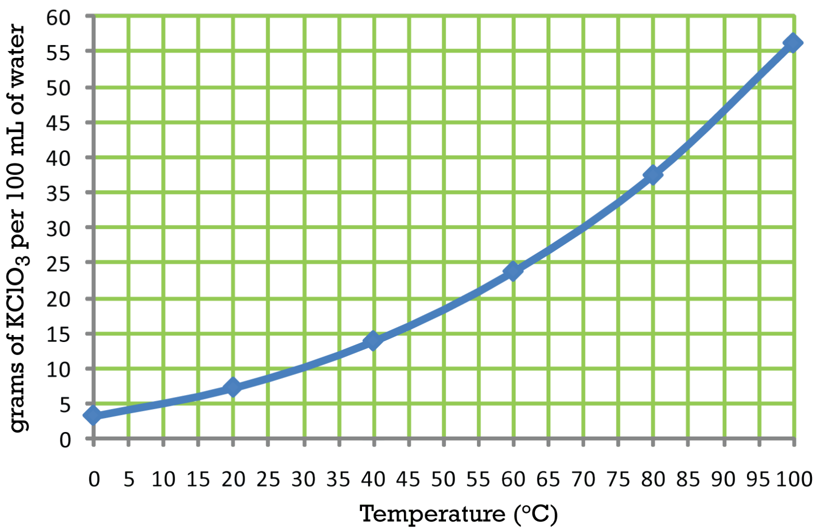
Consider the following set of data for the solubility of $KClO_3$ in water. Data Table B shows that there are exactly six measured data points. When the data is graphed, however, the graph maker assumes that the relationship between the temperature and the solubility exists for all points within the data range. The graph maker draws a line by interpolating the data points between the actual data points. Note that the line is not drawn by just connecting the data points in a connect-the-dot manner. Instead, the line is a smooth curve that reasonably connects the known data points.

Data Table B: Solubility of Potassium Chlorate

Temperature (°C)	Solubility (g/100 mL H_2O)
0	3.3
20	7.3
40	13.9
60	23.8
80	37.5
100	56.3

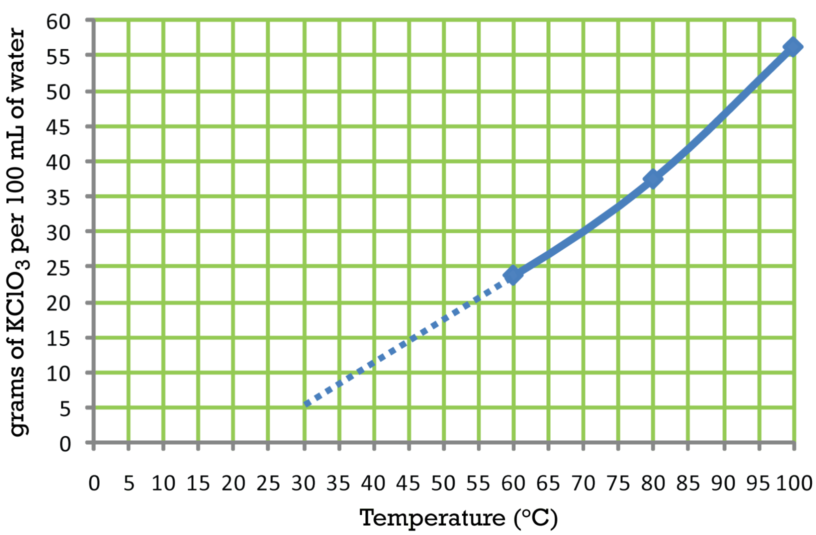
We can now read Graph B1, shown below, for points that were not actually measured. If we wish to determine the solubility of $KClO_3$ at 70°C, we follow the vertical grid line for 70°C up to where it touches the graphed line and then follow the horizontal grid line to the axis to read the solubility. In this case, we would read the solubility to be 30.0 g/100 mL of H_2O at 70°C.

Graph B1: Solubility of Potassium Chlorate



There are also occasions when scientists wish to know more about points that are outside the range of measured data points. Extending the line graph beyond the ends of the original line, using the basic shape of the curve as a guide, is called **extrapolation**.

Graph B2: Solubility of Potassium Chlorate



Suppose the graph for the solubility of potassium chlorate has been made from just three measured data points. If the actual data points for the curve were the solubility at 60°C, 80°C, and 100°C, the graph would be the solid line shown in Graph B2 above. If the solubility at 30°C was desired, we could extrapolate the curve (the dotted line) and obtain a solubility of 5.0 g/100 mL of H_2O. If we check the more complete graph above (Graph B1), you can see that the solubility at 30°C is closer to 10. g/100 mL of H_2O. The reason the second graph produces such a different answer is because the real behavior of potassium chlorate in water is more complicated than the behavior suggested by the extrapolated line. For this reason, extrapolation is only acceptable for graphs where there is evidence that the relationship shown in the graph will hold true beyond the ends of the graph. Extrapolation is more dangerous that interpolation in terms of producing possibly incorrect data.

In situations where it is unreasonable to interpolate or extrapolate data points from the actual measured data points, a line graph should not be used. If it is desirable to present data in a graphic form but a line

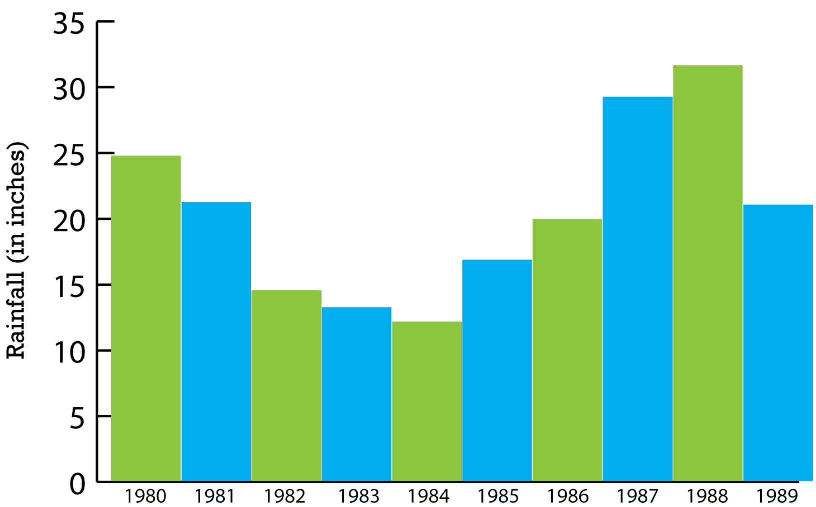
graph is not useful, a bar graph can often be used instead. Consider the data in the following table.

Data Table C: Trout Creek Rainfall 1980-1989

Year	Rainfall (in inches)
1980	24.7
1981	21.2
1982	14.5
1983	13.2
1984	12.1
1985	16.8
1986	19.9
1987	29.2
1988	31.6
1989	21.0

For this set of data, you would not plot the data on a line graph because interpolating between years does not make sense; the concept of the average yearly rainfall halfway between the years 1980 and 1981 would not make sense. Looking at the general trend exhibited by Data Table C also does not provide the slightest amount of evidence about the rainfall in 1979 or 1990. Therefore, the interpolation and extrapolation of the data in this table is not reasonable. If we wish to present this information in a graphic form, a bar graph like the one seen in Graph C would be best.

Graph C: Trout Creek Rainfall 1980-1989



From this bar graph, you could very quickly answer questions like, "Which year was most likely a drought year for Trout Creek?" and "Which year was Trout Creek most likely to have suffered from a flood?"

Finding the Slope of a Graph

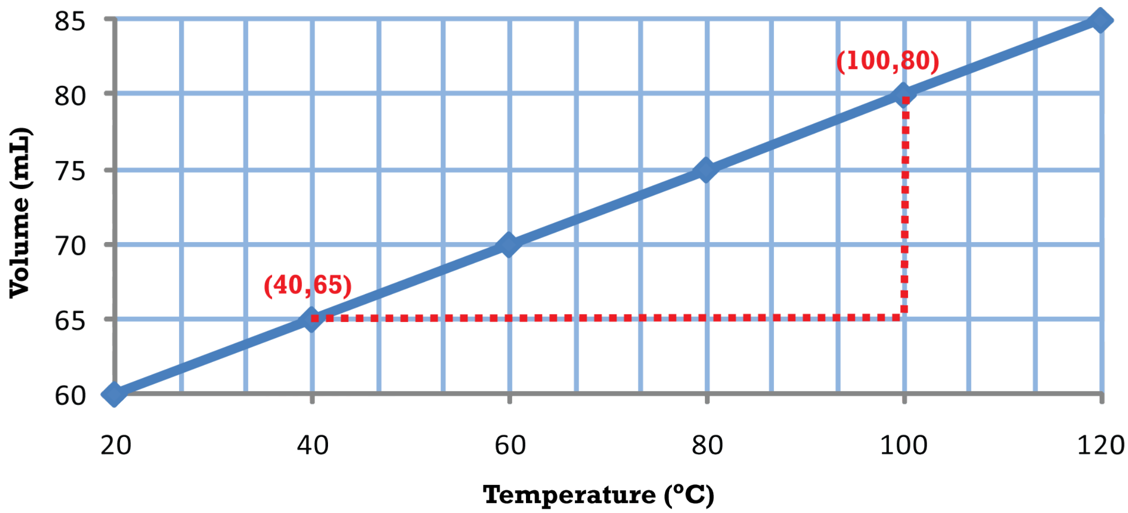
As you may recall from algebra, the slope of the line may be determined from the graph. The slope represents the rate at which one variable is changing with respect to the other variable. For a straight-line graph, the slope is constant for the entire line, but for a non-linear graph, the slope varies at different

points along the line. For a straight-line graph, the slope for all points along the line can be determined from any section of the graph. Consider the following data table and the linear graph that follows.

Data Table D: Temperature vs. Volume Data for a Gas Sample

Temperature (°C)	Volume of Gas (mL)
20	60
40	65
60	70
80	75
100	80
120	85

Graph D: Temperature vs. Volume for a Gas Sample



The relationship in this set of data is linear; in other words, the data produces a straight-line graph. The slope of this line is constant at all points on the line. The **slope** of a line is defined as the rise (change in vertical position) divided by the run (change in horizontal position). For a pair of data points, the coordinates of the points are identified as (x_1, y_1) and (x_2, y_2). In this case, the data points selected are $(40°\text{C}, 65 \text{ mL})$ and $(100°\text{C}, 80 \text{ mL})$. The slope can then be calculated in the following manner:

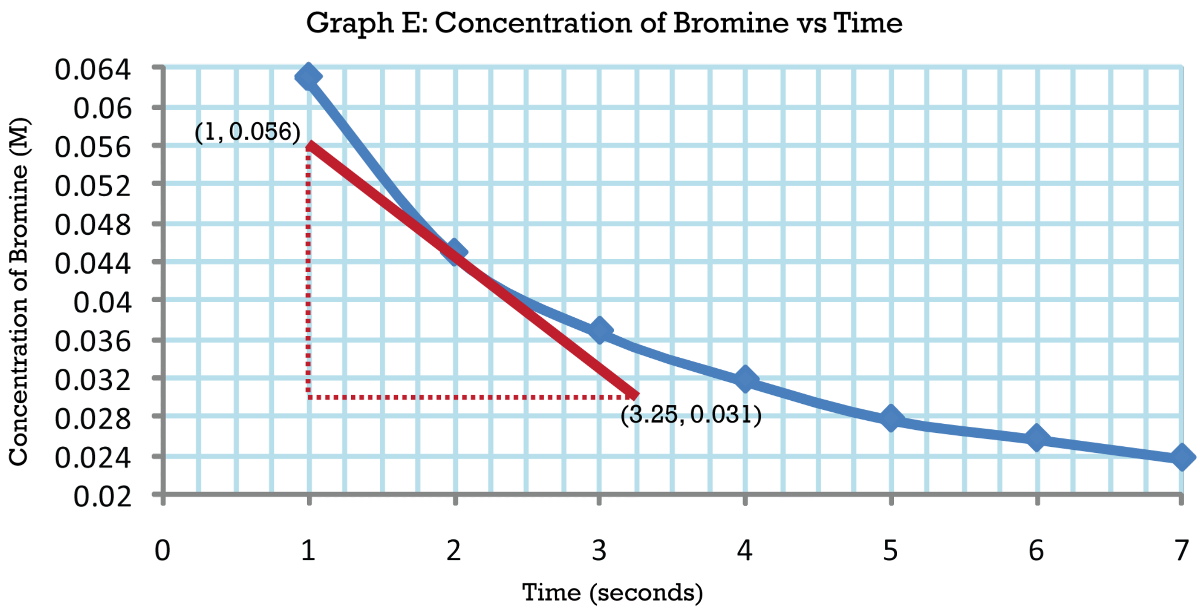
$$\text{slope } m = \frac{rise}{run} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(80 \text{ mL} - 65 \text{ mL})}{(100°\text{C} - 40°\text{C})} = 0.25 \text{ mL}/°\text{C}$$

Therefore, the slope of the line is 0.25 mL/°C. The fact that the slope is positive indicates that the line is rising as it moves from left to right and that the volume increases by 0.25 mL for each 1°C increase in temperature. A negative slope would indicate that the line was falling as it moves from left to right.

For a non-linear graph, the slope must be calculated for each point independently. Since the line will be a curve, the slope is calculated from the tangent to the curve at the point in question. Data Table E and Graph E are for a reaction in which the concentration of one of the reactants, bromine, was measured against time. The concentration is expressed in moles/liter, which is symbolized by M.

Data Table E: Concentration of Bromine vs. Time

Time (seconds)	**Concentration of Br_2 (M)**
1.0	0.063
2.0	0.045
3.0	0.037
4.0	0.032
5.0	0.028
6.0	0.026
7.0	0.024



In order to determine the slope at some point on a curved line, a tangent (approximate) is drawn in as a line that just touches the point in question. Once the tangent has been drawn, the slope of the tangent is determined, which is also the slope of the curve at that point. In the graph above, the tangent has been drawn at the point where t = 2 seconds. We determine the *x*- and *y*-coordinates for two points along the tangent line (as best we can) and use the coordinates of those two points to calculate the slope of the tangent. The coordinates of the point at the left end of the tangent line is determined to be (1.00 s, 0.056 M). The coordinates of the point at the right end of the line is harder to determine, and we are guessing that the coordinates are (3.25 s, 0.031 M).

$$\text{slope } m = \frac{rise}{run} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.031\ \text{M} - 0.056\ \text{M})}{(3.25\ \text{s} - 1.00\ \text{s})} = -0.011\ \text{M/s}$$

Since the slope is a negative number, we know the line is decreasing in height. At t = 2 seconds, the concentration of bromine is decreasing at a rate of 0.011 moles/liter per second. At other points along this curve, the slope would be different. From the appearance of the curve, it is apparent that the slope is negative (the concentration of bromine is decreasing) all along the line, but it appears to be decreasing more quickly at the beginning of the reaction and less quickly as time increases.

Lesson Summary

- Tables and graphs are two common methods of presenting data that aid in the search for regularities and trends within the data.
- When we draw a line graph from a set of data points, we are inferring a trend and constructing new data points between known data points. This process is called interpolation.
- Constructing data points beyond the end of a line graph, using the basic shape of the curve as a guide, is called extrapolation.
- The slope of a graph represents the rate at which one variable is changing with respect to the other variable.
- For a straight-line graph, the slope for all points along the line can be determined from any section of the graph.
- For a non-linear graph, the slope must be determined for each point by drawing a tangent line to the curve at the point in question.

Review Questions

1. What would you do to find the slope of a curved line?
2. Andrew was completing his density lab for his chemistry lab exam. He collected the following data in his data table (shown in **Table** 1.9).
   (a) Draw a graph to represent the data.
   (b) Calculate the slope.
   (c) What does the slope of the line represent?

Table 1.9: **Data Table for Problem 2**

Mass of Solid (g)	**Volume of Solution (mL)**
3.4	0.3
6.8	0.6
10.2	0.9
21.55	1.9
32.89	2.9
44.23	3.9
55.57	4.9

3. Donna is completing the last step in her experiment to find the effect of the concentration of ammonia on the reaction. She has collected the following data from her time trials and is ready for the analysis. Her data table is **Table** 1.10. Help Donna by graphing the data, describing the relationship, finding the slope, and then discussing the meaning of the slope.

Table 1.10: **Data Table for Problem 3**

Time (s)	Concentration (mol/L)
0.20	49.92
0.40	39.80
0.60	29.67
0.81	20.43

Table 1.10: (continued)

Time (s)	Concentration (mol/L)
1.08	14.39
1.30	10.84
1.53	5.86
2.00	1.95
2.21	1.07
2.40	0.71
2.60	0.71

Image Sources

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